

MAS212 Linear Algebra I 2007-2008

Coursework 6

Please hand in your solutions of the **starred** exercises by **2pm on Friday 16 November** using the [blue Linear Algebra I Collection Box](#) on the Ground Floor.

Don't forget to put your **name** and **student number** on your solutions, and to **staple** them.

Exercise* 1. (4 marks) Determine which of the following vectors are linearly independent in $\mathbb{R}^3$:

- (a) $(1, 0, 1)^T, (0, 1, 0)^T, (0, 1, 1)^T$;
- (b) $(1, 0, 1)^T, (0, 1, 0)^T, (0, 1, 1)^T, (3, 2, 1)^T$;
- (c) $(3, 1, 4)^T, (-2, 3, 1)^T, (2, 3, 5)^T$;
- (d) $(3, -3, 1)^T, (2, 0, 2)^T$.

Exercise* 2. (2 marks) Determine which of the following statements (if any) are true. Justify your answers.

- (a) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 3 \\ 3 & -1 \end{pmatrix}$ are linearly independent vectors in $\mathbb{R}^{2 \times 2}$.
- (b) $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$ are linearly independent vectors in P_2 , where

$$\mathbf{p}_1(t) = 1 - t^2, \quad \mathbf{p}_2(t) = t, \quad \mathbf{p}_3(t) = t + 2t^2.$$

Exercise* 3. (2 marks) Show the following:

- (a) If $\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$ is a spanning set for a vector space V and $\mathbf{v}$ is any vector in V , then $\mathbf{v}, \mathbf{v}_1, \dots, \mathbf{v}_n$ are linearly dependent.
- (b) If $\mathbf{v}_1, \dots, \mathbf{v}_n$ are linearly independent vectors in a vector space V , then $\mathbf{v}_2, \dots, \mathbf{v}_n$ cannot span V .

Exercise 4. Let $\mathbf{x}_1, \dots, \mathbf{x}_k$ be linearly independent vectors in $\mathbb{R}^n$, and let $A \in \mathbb{R}^{n \times n}$ be invertible. Define $\mathbf{y}_i = A\mathbf{x}_i$ for $i = 1, \dots, k$. Show that $\mathbf{y}_1, \dots, \mathbf{y}_k$ are linearly independent.

Exercise* 5. (4 marks) For each of the collection of vectors in Exercise 1, decide whether they form a basis for $\mathbb{R}^3$. Justify your answer.

Exercise* 6. (4 marks) Let S be the subspace of $\mathbb{R}^4$ given by

$$S = \{ (r - 2s, r + s - t, s, t)^T \mid r, s, t \in \mathbb{R} \}.$$

Find a basis for S and determine $\dim S$.

Exercise* 7. (4 marks) Find a basis for $S = \{ A \in \mathbb{R}^{2 \times 2} \mid A \text{ is symmetric} \}$ and determine $\dim S$.

Exercise 8. Let S be the subspace of P_3 consisting of all polynomials $\mathbf{p} \in P_3$ with $\mathbf{p}(-1) = 0$. Find a basis for S and determine its dimension.

A cash prize is offered for the best solution received to the following question. Do not hand it in with your coursework, but hand it to me in person either at the Friday lecture or before.

Prize question. A matrix is said to be **anti-symmetric** if $A^T = -A$. Determine the dimension of $S = \{ A \in \mathbb{R}^{n \times n} \mid A \text{ is anti-symmetric} \}$.